MATH 120A Prep: Equivalence Relations

Facts to Know:

Equivalence Relation: An equivalence relation $\sim$ on a set S satisfies three properties:

- Reflexive:
- Symmetric:
- Transitive:

Equivalence Class: The equivalence class of an element x is

Connection to Partitions:

- Equivalence Relation to Partition:
- Partition to Equivalence Relation:

Examples:

1. Define a relation on the set of people where $A \sim B$ if the age of person A equals the age of person B. Show this is an equivalence relation.

2. Show that the relation on $\mathbb{Z}$ defined by $x \sim y$ whenever $3|(x-y)$ is an equivalence relation. What are the equivalence classes?

3. Let $S = \{(x, y, z) \in \mathbb{R}^3 : x, y, z \text{ are 0 or 1}\}$. Define a relation on S by $(x, y, z) \sim (u, v, w)$ if $x + y + z = u + v + w$. Show this is an equivalence relation and write out the equivalence classes.

4. Suppose we want an equivalence relation so that the classes are the lines of slope 5 in $\mathbb{R}^2$. How should we define the equivalence relation?